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Optimal Design of Plug Nozzles and Their Thrust Determination at Start

A. N. Kraiko,* N. I. Tillyayeva,† and S. V. Baftalovskii‡
Central Institute of Aviation Motors, 111250, Moscow, Russia

The design of altitude compensating two-dimensional plug (spike) nozzles, which provide maximum thrust in vacuum, is performed for given values of plug length, "half maximal cross section," and other parameters. The plug is optimally adjoined to the base or to the contour of the turbopump nozzles, turbonozzles, replacing part or of the entire base. At takeoff the thrust of the configurations considered is determined accounting for the boundary-layer separation initiated by the supercritical pressure ratio at the intersection point of the suspended shock with the plug. Computation of separation is performed by using the empirical dependence of the "critical" pressure ratio on Mach number.

Nomenclature

a	=	sound velocity
HY	=	half-base cize
h	=	primary nozzle throat height
k	=	specific heat ratio
L	=	relative length, $L = X/Y$
M	=	Mach number
m	=	flow rate
p	=	pressure
p^+	=	base pressure
R	=	gas constant
$\boldsymbol{R}$	=	thrust
T	=	temperature
$\boldsymbol{V}$	=	velocity vector, $V = \lvert\boldsymbol{V}\rvert$
X	=	plug length
x, y	=	Cartesian or cylindrical coordinates
Y	=	half maximal cross-section plug
α	=	Mach angle, $\sin\alpha = 1/M$
ϑ	=	velocity angle to the x axis
ν	=	0 (1) in two-dimensional (axisymmetrical) case
ρ	=	density

Subscripts

$a, b, \ldots$	=	in points $a, b, \ldots$
st	=	stagnation parameter
t	=	turbopump gas
0	=	prime nozzle
*	=	critical value

Introduction

THE quest for reusable launch vehicles (RLVs) brought to light the problem of two-dimensional plug nozzles.[1,2] The optimal design and thrust determination for plug nozzles in off-design conditions became topical. Besides setting up the conditions for the engine's operation, dimensions, etc., additional requirements must be considered to design a complete vehicle. Thus, vehicles require not only a basic plug nozzle but a turbonozzle (TN) as well. Maximum thrust in vacuum must be provided by both optimally conjoined nozzles. If the aircraft flight in space and takeoff are enabled by the same engines, the nozzle must be altitude compensating. This specifies the choice of a plug nozzle configuration with an incline to its plane of symmetry.

In a RLV the maximum cross section of the aircraft is defined by the distance between two rows of the liquid-propellant rocket engines (LPRE) located on both sides of the plug. The plug length is comparable with the maximal cross-section size. The benefit of this construction is discussed in Refs. 3 and 4. Kraiko and Pudovikov show that at hypersonic flight speed for given aircraft volume and relatively small length minimum drag is achieved by the configuration having a base, located after the maximal cross section of aircraft. For LPREs placed in the afterbody, rigid limitations on the nozzles' block length results in a big plug base. The optimal base size increases with the base pressure p^+. The latter can be increased by bleeding the flow from the turbopump ("turbogas") and a certain amount of flow directed from the main engines (in Ref. 1 it is specified as 1%) through the base. Nevertheless, even when $p^+ = 0$ the system providing maximal thrust for the conditions described has a base. Its size is defined by the so-called "Busemann condition."[5] This condition also defines optimal conjunction of the plug and TN contours. They both are designed by means of the solutions of "external" and "internal" variation problems of supersonic gasdynamics.[5] Finding a solution to the mentioned problems has a long history in Russia, only partially known in the West. This history is summarized next.

Most of the widely used results concerning the derivation of necessary conditions for the determination of optimal nozzles shapes were obtained by means of the control contour method (CCM). The first to implement a transfer to characteristical control contour (CC) was Nikolskii.[6] In 1950 he solved the problem of fore- and afterbody design for ducted bodies, which display minimal wave drag at supersonic flow, using linear approximation. In 1955 Guderley and Hantsch,[7] independently, used a transfer to characteristical CC for solving the maximal thrust design problem of supersonic parts of two-dimensional and axisymmetrical nozzles in its accurate formulation. For an irrotational (homogeneous in stagnation parameters) flow the desired optimal contour must provide the maximal nozzle thrust $\boldsymbol{R}$ at given subsonic nozzle contour to the left from point a (Fig. 1a), the supersonic section length X, and the endpoint of nozzle ordinate Y or external pressure p^+. If there are no constraints on contour's curvature, it shows a discontinuous slope in point a, overflowed with formation of C^--characteristics rarefaction waves fan. In CCM the problem of the optimal contour ab design is reduced to a one-dimensional variational problem of the optimal C^+-characteristic cb determination. When sections of the closing C^- characteristic of the fan ac and the optimal C^+ characteristic cb are determined, the contour ab is designed as a streamline from the corresponding Gursat's problem solution.

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*Chief of Department, Aviamotornaya 2.

†Senior Research Scientist, Aviamotornaya 2.

‡Research Scientist, Aviamotornaya 2.

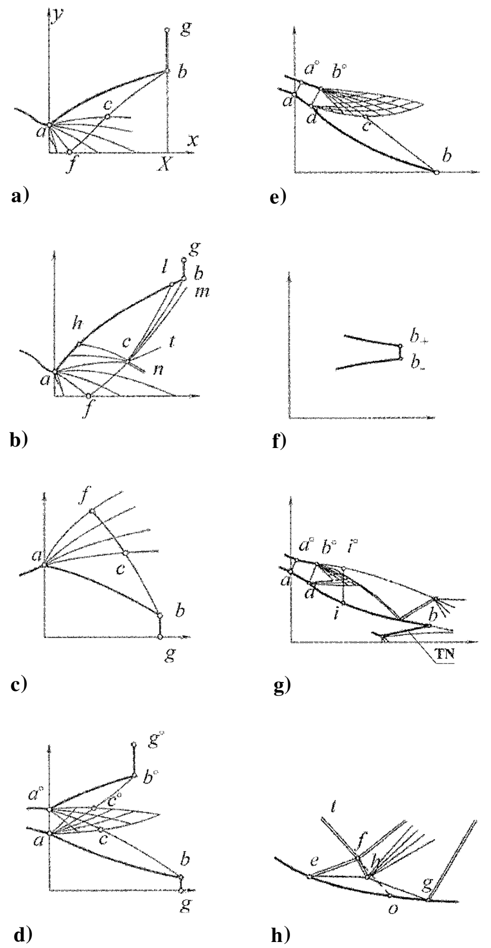

Fig. 1 Schemes of optimal configurations and flows.

In Ref. 7 the design of optimal C$^+$-characteristic cb was reduced to the solution of a boundary problem for ordinary differential equations with given boundary conditions in points b and c. Besides, it was established that flow parameters are constant on cb for optimal two-dimensional nozzle, and at unfixed ordinate y_b the same condition for two-dimensional and axisymmetrical cases is satisfied in point b. This condition for determining y_b is named as Busemann's condition (BC) in Ref. 7. BC for the problem discussed named further as an internal one and an analogous condition for a further discussed external problem take the following form:

$$P^{\pm}_{b\pm} - p^+ = 0$$

$$P^{\pm} = p \pm \rho V^2 \tan\alpha \sin\vartheta \cos\vartheta = p\left(1 \pm \frac{\kappa M^2}{2\sqrt{M^2-1}}\sin 2\vartheta\right) \qquad (1)$$

Here the second expression for $P^{\pm}$ is written for a perfect gas, $b_{\pm}$ is the contour's endpoint, and gas parameters are marked with the index $b\pm$ to the left from point $b_{\pm}$ on the section, overflowed by a supersonic flow. Upper (lower) signs and indexes correspond to the internal (external) problem.

In 1957 Shmyglevskii[8,9] integrated the just-mentioned boundary problem (see also later publications in Refs. 10 and 11) and obtained two finite relations for determination of parameters' distributions on the optimal characteristic cb. These relations were written in Refs. 8–11 for a perfect gas with a constant ratio of specific heat k. In the same year Sternin[12] obtained analogous relations for arbitrary two-parameter gas using the same technique. For two-dimensional and axisymmetrical cases those relations take the following form:

$$V\cos(\vartheta-\alpha)/\cos\alpha = \text{const}, \qquad y^{\nu}\rho V^2 \sin^2\vartheta \tan\alpha = \text{const} \qquad (2)$$

The preceding formulas first obtained in Ref. 12, were published in the periodical scientific press only in the beginning of 1959.[13] A bit earlier, in the end of 1958, Rao[14] published the same results. In his CCM cb was not assumed to be a section of C^+characteristic from the very beginning. This circumstance led to the criticism[15] that the final result in Ref. 14 was casual because it was obtained by means of a erroneous method. Later, arguably as a result of that criticism, Rao revised the work.[16] The approaches of Refs. 9 and 10 were practically repeated for solving the problem, differing only in the solution "guessing" and in a series of incorrectly presented details (the latter are discussed in the Russian translation.[17]. A noncontradictional interpretation of "noncharacteristical" CCM version[5] was given in 1979 by one of the present report's authors. This version of the method, named in Ref. 5 as indefinite control contour method (ICCM), provides nowadays the most simple way for obtaining the necessary optimal conditions in problems, which admit transfer to CC.

In Refs. 8–10 Shmyglevskii considered, along with irrotational flows with constant stagnation parameters, a case where those parameters are not constant on different streamlines at the nozzle inlet. In 1959 Guderley[18] made an analogous generalization for a gas, insignificantly differing from perfect. Generalization for an arbitrary two-parametric gas, which leads to the second formula from Eq. (2) also, holding true for a rotational (unhomogeneous in stagnation parameters) flow, is given in Ref. 19. Optimal solutions with continuous parameters' distributions on the characteristic fcb gained the name of "continuous."

In 1961 Sternin[20] found out that the continuous solutions can be designed not in any choice of point c of the initial rarefaction wave fan. After that, in 1962 Shmyglevskii obtained the necessary conditions of maximal thrust in the form of inequalities. For these cases he designed the "discontinuous" optimal solutions[21] with C$^-$characteristics focusing in point c (Fig. 1b where ach and lcm are the compression and rarefaction fans, cn is the shock wave, and ct is the slip line, respectively). In further investigations[10,19] of discontinuous solutions, it was established that the same conditions as for continuous solutions hold at cb [relations (1) for irrotational flows], and two more conditions of transversality appear in point c. The investigations about the design of maximal thrust conventional nozzles supersonic parts have been completed in Ref. 22. Particularly, in this investigation a special role of the base bg, where $x = X$ is emphasized. When apart from X, a Y ordinate of the contour endpoint is given, then the base can appear as a section of boundary extremum caused by limitations of the maximally permitted nozzle length. The base bg with $y_b < y_g = Y$ is not overflowed by the supersonic flow, and the value y_b is determined by the BC. Included in BC the base pressure p^+, acting on the base, is supposed to be given and independent from the form of the section ab.

Occurance of a base in an isolated nozzle's optimal contour at given X and Y evidences that the given ordinate Y is overdimensioned and needs reduction. This is the reason why the introduction of a base in the problem discussed seems artificial to some extent. To the contrary, configurations with a base, being a section of a boundary extremum, are not only natural, but even typical[23,24] when the nozzle and vehicle's outer contours are designed jointly. It will be further shown that a base is a rather widespread element of optimal contours of altitude-compensating plug nozzles.

The just discussed variational problems of supersonic gasdynamics gained the name of internal. In 1957 Shmyglevskii[8,9] solved analogous external problems for rotational and irrotational flows of a perfect gas at the same time as the internal ones (see also Refs. 9 and 10; generalization for an arbitrary two-parametric gas is given in Ref. 19). The difference between the external and internal problems is caused by the fact (Fig. 1c) that supersonic flow is limited by the contour ab from below in the external problems and from above- in the internal ones. According to this fact, cb is here a section of C$^-$characteristic. Instead of Eq. (2) the following relations are satisfied on cb:

$$V\cos(\vartheta+\alpha)/\cos\alpha = \text{const}, \qquad y^\nu \rho V^2 \sin^2\vartheta \tan\alpha = \text{const} \tag{3}$$

The first expression holds true only for irrotational flows. The expressions (3) were obtained by Rao in 1961[25] just for irrotational flows when solving the optimal design problem of nozzle with a plug. But configurations, considered by Rao as well as in Refs. 26 and 27 in its approximate formulation, had no slope discontinuity in the plug's contour. They form a narrow class of nozzles, which are optimal only in specific situations. In the general case the designed and given sections of the plug's contour are conjoined with a discontinuous slope or with a section of maximal acceptable (in absolute value) curvature. Previously, such configurations were considered in Refs. 28 and 29. In Ref. 28 the upper-wall supersonic section $a^\circ b^\circ$ of a given length was designed along with a plug (Fig. 1d). This example demonstrates using expressions (2) and (3) for a single device optimal design. The same opportunity is used further in a slightly different way.

The review of solving the supersonic gasdynamics variational problems in their accurate formulations is not complete. Additional information on the results, obtained up to 1979, can be found in review[30] and monograph.[5] Among other subjects, the development and application of the general method of Lagrange multipliers (MLM)[31] and nozzles' design for nonequilibrium and two-phase flows are considered.[5] After 1979 new results were obtained, for example, on designing nonsymmetrical two-dimensional nozzles, for which ICCM allows incorporating in the problem not only thrust but also lift and momentum.[32,33] Significant progress is achieved in solving the design problem of the whole nozzle, not only of its supersonic section.[34] An exhaustive analysis is carried out for optimal leading sections, overflowed with an attached shock wave.[35] The problem of designing nonconventional nozzles with big turn angles of supersonic flow[36] is solved in terms of MLM.

Results

As just mentioned, for a general case the supersonic plug contour adjoins to the primary nozzle contour with a discontinuous slope (or with a section of maximally permitted curvature). The overflow of the discontinuous slope contour (Fig. 1e) is accompanied by a formation of C$^+$-characteristics rarefaction fan. As a result, the flow acceleration is realized not in one fan around an inclined outer lip, but in two expansion fans. It allows designing optimal plugs of any length and degree of expansion at initial flow angles $\vartheta_0 < 0$. The designed initial section of optimal plug and primary two-dimensional nozzle with a uniform flow on its exit are shown in Fig. 2 together with C$^+$- and C$^-$-characteristics fans, starting from the plug slope discontinuity point and from the primary nozzle lip. This example corresponds to $X = 270$ and $Y = 100$. Here and further the lower point a of the inclined throat aa° of the equivalent two-dimensional primary nozzle (which in RLV results from uniting axisymmetric nozzles of 49 linearly located LPREs) lies on the y-axis of Cartesian coordinates xy, and its height h is taken as the length scale. Here the specific heat ratio of the primary gas $k_0 = 1.2$, its stagnation pressure $p_{\text{st0}} = 160$ atm, the inclination angle of speed vector to the x axis and Mach number on the exit of the primary nozzle are the following: $\vartheta \equiv \vartheta_0 = -45$ deg and $M \equiv M_0 = 2.5$. The x axis lies in the plug plane of symmetry.

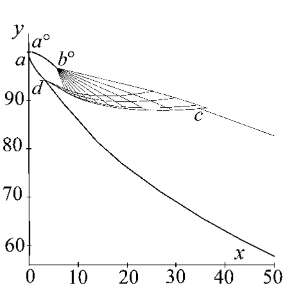

Fig. 2 Primary nozzle, initial part of optimal plug, and characteristics fans.

When the plug and TN are designed together, not only the primary gas parameters but also k_t, $(p_{\text{st}})_t/p_{\text{st0}}$, $(RT_{\text{st}})_t/(RT_{\text{st0}})$ and m_t/m are specified:

$$k_t = 1.37, \qquad (p_{\text{st}})_t/p_{\text{st0}} = 0.0158$$

$$(RT_{\text{st}})_t/(RT_{\text{st0}}) = 1.179, \qquad m_t/m = 0.0417$$

The optimal plug contour or, more precisely, optimal distributions of parameters on the section cb of C$^-$-characteristic $b^\circ b$ in Fig. 1e and the ordinate value of point b or b_+ are found by solving the external variational problem of supersonic gasdynamics. According to this solution, maximal thrust is provided by constant parameters on cb.

End plug point b_+ may not coincide with the end point b_- of TN contour, as shown on Fig. 1f. In that case y_{b+} and y_{b-} are determined by the Busemann condition (1) with a corresponding sign and index. Base pressure p^+ included in conditions (1) is considered to be known and independent from the plug and TN contours shapes. In fact, it is not so for a base, overflowed on both sides by supersonic flow, which is formed by the plug and TN. It is not essential, however, at $X = 270$, $Y = 100$ and chosen parameters on the exit of the primary nozzle and TN. Under these conditions and $p^+ < \max(p_{b+}, p_{b-})$ the ordinate value y_{b+} is smaller than ordinate y_{b-}. Therefore, a configuration without a base is optimal, and the value of $y_{b+} = y_{b-} = y_b$ is determined by the equality: $P^+_{b+} = P^-_{b-,}$ where $P^\pm$ are taken from Eqs. (1).

Under the conditions discussed and TN lengths being $X_t = 12$, 21, 25, and 42, four configurations were designed, providing maximal thrust in vacuum. Contours of one of them are shown in Fig. 3, where x and y scales are different.

To determine the capacity of the designed configurations to compensate for additional thrust losses at off-design altitude, their thrust at start was calculated. For this purpose the initial section of the primary nozzle supersonic jet before the section ii° (Fig. 1g) was calculated at external pressure $p^e = 1$ atm by the method of characteristics. Then the calculation was made according to the decay monotonous march scheme of shock-capturing computation for supersonic flow[37]. The authors considered the second-order approximation; however, the use of the first-order scheme for two grids with consequent Richardson extrapolation on zero step proved to be more convenient.

The flow for one of the designed plugs at start is shown in Fig. 4a. In this picture the Mach numbers field and the discontinuity of the jet boundary slope reveal the existence of a suspended shock, reflecting from the plug at $x \approx 92$, and a rarefaction fan, starting from the point, where the shock, reflected from plug, meets the jet's boundary. The shocks are drawn with double lines in Fig. 1. In this example the pressure ratio in the point where the suspended shock meets the plug exceeds the critical one, which is calculated for turbulent boundary layer by the empirical formula[38]:

$$p_{\text{cr}}/p = 1 + 0.2kM^2/(M^2-1)^{\frac{1}{4}} \tag{4}$$

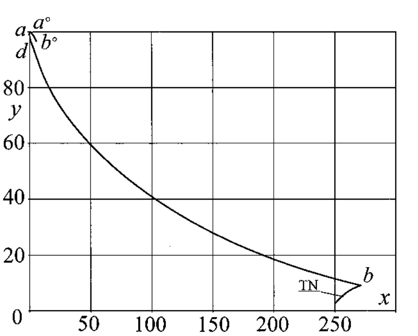

Fig. 3 Contours of primary nozzle, optimal plug, and turbonozzle (*L* = 2.7).

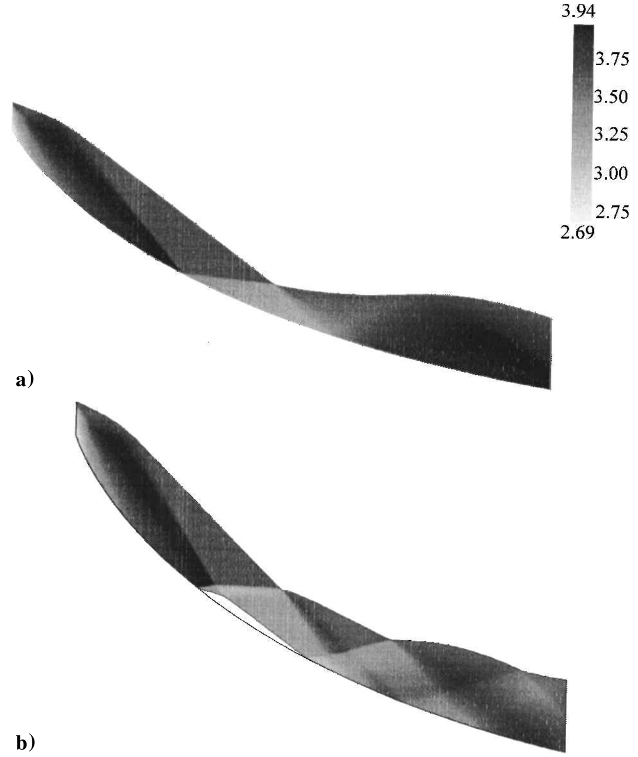

Fig. 4 Mach-number distributions in jet (L = 2.7, *a*-unseparated regime, *b*-separated regime).

Table 1 Plug nozzles characteristics for L = 2.7

NN	y_b	$\boldsymbol{R}$	$\boldsymbol{R}_t 10$	$\boldsymbol{R}_\Sigma$	$\boldsymbol{R}^{id1}$	$\Delta\boldsymbol{R}_1$	$\boldsymbol{R}^{id2}$	$\Delta\boldsymbol{R}_2$
1v	10.8	2.8579	0.998	2.9577	3.4335	13.9	2.9702	0.4
1e	10.8	2.4551	0.551	2.5102	2.5584	1.9	2.5584	1.9
1e 80	10.8	2.4557	0.551	2.5108	2.5584	1.9	2.5584	1.9
2v	9.7	2.8589	0.988	2.9576	3.4335	13.9	2.9702	0.4
2e	9.7	2.4524	0.551	2.5075	2.5584	2.0	2.5584	2.0
2e 85	9.7	2.4534	0.551	2.5085	2.5584	2.0	2.5584	2.0
2e 75	9.7	2.4536	0.551	2.5087	2.5584	1.9	2.5584	1.9
3v	9.1	2.8598	0.980	2.9578	3.4335	13.9	2.9702	0.4
3e	9.1	2.4519	0.551	2.5070	2.5584	2.0	2.5584	2.0
3e 80	9.1	2.4531	0.551	2.5082	2.5584	2.0	2.5584	2.0
4v	7.4	2.8606	0.956	2.9562	3.4335	13.9	2.9702	0.5
4e	7.4	2.4490	0.551	2.5041	2.5584	2.1	2.5584	2.1
4e 80	7.4	2.4484	0.551	2.5035	2.5584	2.1	2.5584	2.1

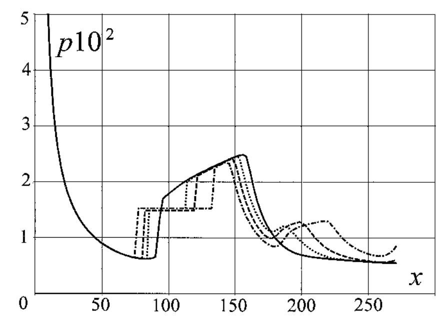

Fig. 5 Pressure distributions along the plug (L = 2.7) at start.

Here M and p are the values before the shock, and $p_{\rm cr}$ is the critical pressure value on the shock. This means that instead of an attached flow, resulting from the ideal-gas approximation, a more complicated flow with separated zone is observed. At first sight Reynolds equations with corresponding closure equation are necessary to compute this flow. Such closure equations, however, are actually not more reliable than formula (4) for separations induced by interaction of shocks with a turbulent boundary layer. It will be demonstrated further that this formula allows to determine the plug's contribution into the thrust with a rather high precision.

The separated flow scheme, taking place in such a case, is shown in Fig. 1h, where ehg is the separated zone boundary, tf the suspended shock, and ef the shock on which the critical pressure ratio determined by formula (4) is observed in the point e. In terms of the scheme in Fig. 1h, the same pressure is observed in the separation zone under the streamline ehg. Hence, this leaves only one uncertainty for the calculation of the separated flow, which is the coordinate of point e on the plug. To avoid it, the experimental dependency[38–40] analogous to Eq. (4) can be used for nondimensional "separation distance" $(x_o - x_e)/\delta_o^*$ computation. In this dependency all parameters, including the displacement thickness of boundary layer δ_o^*, are taken in point o, where the suspended shock meets the plug for its attached flowing by the ideal gas. The calculation of a boundary layer in the accelerating flow primary nozzle and on a plug before the point o is not a problem. Nevertheless, it also proved to be unnecessary for determining the nozzles' thrusts at off-design conditions. The accuracy of such a conclusion is proved by comparison of thrust determined by formula (4) for $x_e = 75$, 80, 85, and 90 at $x_o \approx 92$.

Computational characteristics are shown in Table 1, Fig. 4b and Fig. 5. The first column in Table 1 outputs the "nozzle name," number, its operational regime ("v"in vacuum, "e"on Earth), and value x_e (numbers 75, 80, and 85; their absence means that the flow is attached). Figure 4b corresponds to the separated flow at $x_e = 75$. In Fig. 5 solid curves represent pressure distributions along the plug for the attached flow and other curves for the separated flow. The separation zones size on them is determined by the length of horizontal parts. Figures 4 and 5 demonstrate that the choice of the point e position on the plug has a great influence on the flow picture and the distribution of p. Nonetheless, as can be seen from the Table 1, the thrusts values $\boldsymbol{R}$ and $\boldsymbol{R}_\Sigma = \boldsymbol{R} + \boldsymbol{R}_t$ are practically independent from the separation zone's size. Here $\boldsymbol{R}$ is the sum of x components of the primary nozzle's thrust and the acting to plug pressure forces integral. By virtue of formula (4) and because of the flow separation immediately after the angle discontinuity point in TN's throat, the optimal TN operates at start practically without overexpansion and related with it losses $\boldsymbol{R}_t$. In Table 1 the values of ideal thrust ($\boldsymbol{R}^{\rm id}$) and the losses of $\boldsymbol{R}_\Sigma$ in percents ($\Delta\boldsymbol{R}$), corresponding to gas expansion up to external pressure (or area Y), are marked with index 1 (2). Thrusts $\boldsymbol{R}$, $\boldsymbol{R}_t$, and $\boldsymbol{R}_\Sigma$ are given for half of devices and are scaled by $\rho_* a_*^2 h$, with dimensional values of critical density ρ_* and critical speed a_* on the entrance to the primary nozzle and its dimensional throat size h.

Unlike the results just presented for a long plug ($L = 2.7$), the following examples correspond to an RLV with a short plug. The peculiarity of the engine is the base, where size $2y_b$ is comparable with X and Y. A big base results when the value of L has an order of one. Then for acceptable values of ϑ_0 and M_0 for primary nozzles, the condition (1) leads to the difference of ordinates $y_{a^\circ} - y_b < Y$ even at $p^+ = 0$. Thus, for cases just considered determined by the difference $y_{a^\circ} - y_b$ an optimal expansion ratio of the plug for $p^+ = 0$ equals 112. That is why the optimal plug will have a base when $L = < 270/112 \approx 2.4$. For $p^+ > 0$ the optimal base size exceeds the corresponding value for $p^+ = 0$. Really $p^+ > 0$ even during flight through the vacuum caused by partial "screening" of the base region by supersonic flows overflowing the plug and caused by the inflow of turbogas and of a small amount of primary engine gases. The incompleteness of screening is caused by the existence of the side boundaries in the base region. Gas outflow into the surrounding space reduces p^+ and, at the same time, makes the problem of accurately determining p^+ more complicated.

In the RLV[1,2,41] $L \approx 1.35$. Because of the lack of a certain information on other parameters, three values of h were chosen. They correspond to minimum, maximum, and intermediate values of h within the limits of available data about the RLV . For these h values the following plug lengths and half-maximal crosssection in ascending order of h were obtained: $X_1 = 368$, $Y_1 = 273$; $X_2 = 279$, $Y_2 = 207$; and $X_3 = 233$, $Y_3 = 173$. Further, the characteristics of corresponding configurations are marked with indexes 1, 2, and 3.

Optimal plug contours, designed for $p^+ = 0$, have relative base sizes $H_1 = 0.47$, $H_2 = 0.44$, and $H_3 = 0.43$, where H is scaled by Y. The base size in the RLV is smaller ($H = 0.37$) in spite of the intentions[1] to obtain comparatively high values of p^+ at the expense of gas inflow into the base region. The H values just presented slightly decrease with the increase of h. The change of M_0 from 2.5 to 2 has also a small influence, and H_1 at such a change increases up to 0.48. Solely as a result of gasdynamic thrust optimization in RLV, the base size is undervalued. Perhaps the decrease of H provides a higher p^+, reducing the gas leakage from the base region through its side boundaries. A strong relation of p^+ and the base size during the injection into the base region leads to such a modification of the Busemann condition[19] that it will give a smaller H than obtained in Eqs. (1). Independently, according to Refs. 39 and 40, the optimization not by thrust but by its relation to the device weight leads to a significant reduction of the size and the weight of the base (in Refs. 41 and 42 by 9.4% and 18.7%, respectively) at almost constant thrust (within 0.1%). It seems that this is exactly what caused a value of H in RLV, smaller than the one just obtained.

Contours of three plugs are shown in Fig. 6, where x° and y° are coordinates x and y scaled by X. The distribution of p along one of the plugs at start is shown in Fig. 7, analogous to Fig. 5, and characteristics of all designed configurations are gathered in Table 2, analogous to Table 1. Despite the noticeable difference in plugs' lengths and expansion rates, the values of $\boldsymbol{R}$ proved to be similar for all cases.

Table 2 Plug nozzles characteristics for $L \approx 1.35$

NN	X	Y	y_b	$\boldsymbol{R}$	$\boldsymbol{R}^{id1}$	$\Delta\boldsymbol{R}_1$	$\boldsymbol{R}^{id2}$	$\Delta\boldsymbol{R}_2$
1v	367.6	272.5	127.8	2.8901	3.3156	12.8	2.9673	2.6
1e	367.6	272.5	127.8	2.4301	2.5030	2.9	2.5030	2.9
1e 80	367.6	272.5	127.8	2.4215	2.5030	3.3	2.5030	3.3
1e 90	367.6	272.5	127.8	2.4265	2.5030	3.1	2.5030	3.1
2v	279.0	207.2	92.4	2.8693	3.3156	13.5	2.9459	2.6
2e	279.0	207.2	92.4	2.3949	2.5030	4.3	2.5030	4.3
2e 90	279.0	207.2	92.4	2.3942	2.5030	4.3	2.5030	4.3
3v	233.4	173.0	73.9	2.8551	3.3156	13.9	2.9309	2.6
3e	233.4	173.0	73.9	2.4456	2.5030	2.3	2.5030	2.3
3e 90	233.4	173.0	73.9	2.4473	2.5030	2.2	2.5030	2.2

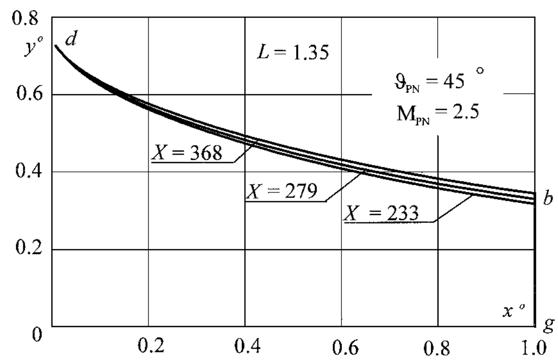

Fig. 6 Optimal contours of short plug ($L \approx 1.35$).

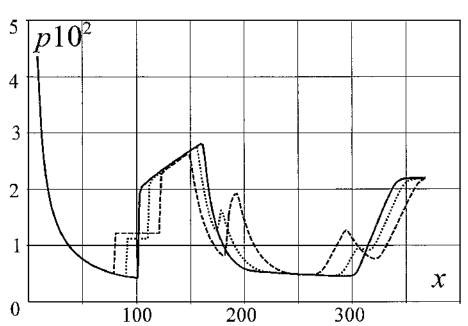

Fig. 7 Pressure distributions along the plug ($L \approx 1.35$) at start.

Rather higher values of $\Delta\boldsymbol{R}_1$ at start and $\Delta\boldsymbol{R}_2$ at all regimes are observed for these configurations, but like in Table 1 $\boldsymbol{R}$ is weakly related to the separation point coordinate and to significantly different attached and separated p distributions along the plug. Even when the small injection throughout the base exists at start, the ejecting effect of primary jets will cause a rarefaction in the base region ($p^+ < p^e$) and additional losses. To the contrary, the contribution of a nonzero p^+ into the thrust in vacuum will be positive. This is what the designers of RLV probably count on.

To get an estimate of the effect caused by $p^+ > 0$, we will present the values of M_b, p_b, and ϑ_b for one of the plug in Fig. 6. They are the following: $M_{b1} = 4.8$, $p_{b1} = 0.0011$, and $\vartheta_{b1} = -9.9°$. If we convert from these values to the equivalent ones,[43] corresponding to $\vartheta_b = 0$, we will have $M'_{b1} = 4.3$, $p'_{b1} = 0.003$. In the real jet cross section $x = x_b$ the greater is the distance from the point b, the Mach numbers and ϑ increase, and the pressure decreases. Accordingly,[43] such nonuniformity reduces p^+ in comparison with the value, obtained for a uniform flow with $M \equiv M_b$, $p \equiv p_b$, and $\vartheta \equiv \vartheta_b$. A significant decrease of p^+ should be expected also because of the just-mentioned side outflow from the base region. For a flow with $\vartheta \equiv 0$ and $M \equiv M'_b \approx 4$ without side outflow from the base region,[41] $p^+ = (0.15 \div 0.2)p'_{b1}$. If $p^+ = 0.1p'_{b1}$ is taken, accounting for the p^+ just listed reducing effects, then $\chi^+ \approx 0.04$ is added to the vacuum thrust for the designed configurations. This will cause a 1% reduction of $\Delta\boldsymbol{R}_1$. It must be, however, taken into consideration that in RLV[1] the plug width equals only to seven heights of its base. At such base region height and width the influence of the side outflow with a close-to-sonic speed can be rather strong. In any case, when the values of p^+ are different from $0.1p'_{b1}$ the presented data allow easily the recalculation of the base contribution into $\boldsymbol{R}$, $\Delta\boldsymbol{R}_1$, $\Delta\boldsymbol{R}_2$. Accurate determination of vacuum thrust for configurations with a big base requires the consideration of the influence on base pressure of the side outflow from the base region, and also a taking into account the possible decrease of such an outflow by means of the base screening by jets, flowing from the two rows of turbonozzles, located along the edges of the base cut.

Thrust losses of the plug nozzles at start are significantly lower than those of the "bell" axisymmetric nozzles of the same expansion ratio, optimal in vacuum. In typical cases they decrease three to four times (from 11 to 3%).

The approaches just described for design optimal plug systems and their thrust determination under off-design conditions, particularly at start, are applicable also to axisymmetrical plug configurations, which are also of increasing interest.[44,45] Their incorporation into multidisciplinary optimization algorithms is also promising. Such algorithms, based on the direct methods, are developed in connection with the problem of two-dimensional altitude-compensated nozzles RLV design in Refs. 41, 42, and 46.

Conclusions

Considered nozzle configurations differ by values of relative length L. At $L > 2.4$ the plug adjoins to turbonozzle without a base. At $L < 2.4$, as for an "Aerospike"—reusable launch vehicle where $L = 1.35$, the plug has a big base. Using primary nozzles with exit Mach number $M_0 = 2.5$ and inclination angle to the plane of symmetry $\vartheta_0 = 45$ deg provides relatively small thrust losses either in vacuum and at start, ensuring altitude compensation. At start the thrust of designed configurations is determined with accounting for the boundary-layer separation, initiated by the supercritical pressure ratio in the intersection point of suspended shock with the plug. Computation of separation is done by using the empirical dependence of critical pressure ratio on Mach number. By virtue of the same dependence, the optimal TN operates practically without overexpansion at start as a result of separation in its throat. At start the thrust losses of the plug nozzles are three to four times smaller than those of the axisymmetrical "bell" nozzles with the same expansion ratio.

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